What is perspective?

Art of Mathematics, Summer 2024

1 Introduction

Today we'll talk about perspective.

Perspective is a method of representing three-dimensional scenes on a two-dimensional surface. The use of perspective is one quality which makes paintings, drawings, etc. appear as accurate representations of scenes from real life. There is a variety of geometric methods for creating two-dimensional depictions of three-dimensional objects. One type of geometric method, *linear perspective*, seeks to make the relative positions of straight lines appear as they would to the human eye from a certain vantage point. We'll focus on the simplest of this family of methods: *one-point perspective*.

One-point perspective is characterized by the fact that all lines which run parallel to the viewer's fixed line of sight converge to a common point, called the *vanishing point*. If you think of railroad tracks running off from beneath your feet to the distant horizon, you may see in your imagination that the two tracks come together in a faraway point, just where they disappear over the horizon. That far-off point is the vanishing point of this imaginary scene, and the disappearance of the tracks explains the terminology.

This and other geometric methods of perspective have become powerful tools in the hands of visual artists since they began to be formalized during the Italian Renaissance. The ideas also have important applications to graphical representation and imaging in engineering, medicine and many other fields. Just as geometry fed the growth of art, the development of perspective in art inspired discoveries in geometry. The idea that even parallel lines, such as those railroad tracks, might all come together in some infinitely distant point led to the formulation of *projective geometry*.

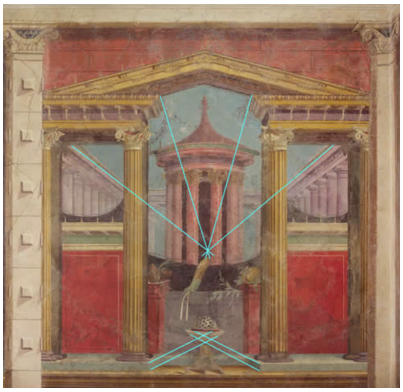
2 History

2.1 Antiquity

Attempts to endow paintings and drawings with a sense of three-dimensionality go back to the very earliest civilizations. Egyptian wall paintings depict figures overlapping and varying in size in order to suggest differences in distance from the viewer. The Greek geometer Euclid surmised that proportion gave the appearance of depth, and by the 1st century B.C.E., ancient Roman artists were employing vanishing points—though not in a totally systematic manner—as seen in the wall of art of the Second Style presevered in the ruins of Pompeii. Classical Greek and Roman techniques would be rediscovered during the Italian Renaissance and inspire the modern mathematical notions of perspective

Chinese landscape painters created a sense of depth with techniques such as making more distant objects less sharp and colorful. As the art historian Erwin Panofsky observed, it may be that the subjects favored

(a) Egyptian wall painting in the New Kingdom. Between 6000 B.C. and 400 A.D.



(b) Villa of P. Fannius Synistor Cubiculum M alcove. Middle of the first century B.C. Pompeii, Italy.

(c) *Looking in a Mirror by an Ornamental Box*, Wang Shên (c. 1036–c. 1093), Southern Sung dynasty.

Figure 1: Perspective in ancient art. [Source: "The History of Perspective," *Essential Vermeer*.]

by East Asian artists provided little motivation to develop methods of linear perspective in comparison with those favored by artists in Europe. The scenes of nature, mythical creatures, the human form and even some Eastern architecture offer few sustained straight lines which the artist would wish to represent in "proper" perspective. Nevertheless, geometric methods of depicting the three-dimensional did emerge in East Asia. By the medieval period, Chinese artists had developed what mathematicians today call the oblique projection. Here lines do not converge toward a vanishing point as they would appear to do in the viewer's eye; rather, lines which are parallel in the three-dimensional scene remain so in the two-dimensional representation.

2.2 Renaissance

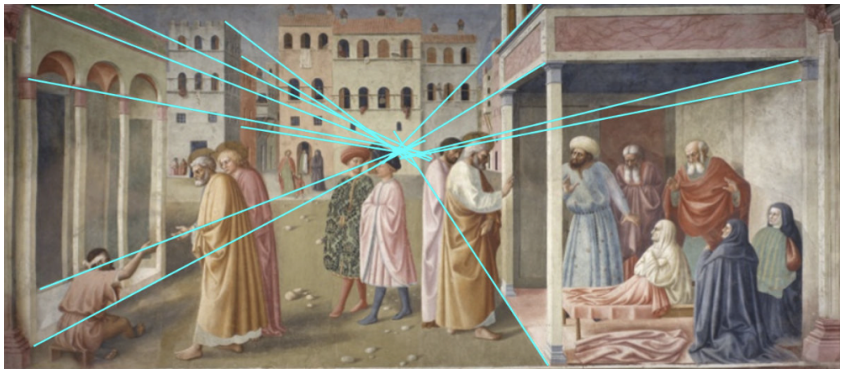
The use of perspective in European art became more sophisticated with the dawn of the Italian Renaissance. Although precise geometric methods of perspective had not been fully worked out, hints of them began to appear in artists' works by the 14th century. True linear perspective was discovered in Italy in the 15th century. Pioneers in its use include Filippo Brunelleschi, Leon Battista Alberti, Masaccio, Paolo Uccello, Piero della Francesca and Luca Pacioli. It is generally agreed that the method of one-point perspective was first worked out by Brunelleschi, based on his study of ancient Roman artwork, in about 1415. This was then mathematically formulated by Alberti in his work *De Pictura* (*On Painting*) in 1435.

One-point perspective rapidly became ubiquitous in the work of Italian Renaissance painters, reaching the height of its sophistication in works by painters like Rafael, Masolino, and Leonardo da Vinci (Figure 2(a)-(c)). As with other discoveries of the Italian Renaissance, the use of one-point perspective moved out of Italy to the rest of Europe gradually over the course of the 15th and 16th centuries. Among the earliest examples of its adoption by a Northern European artist is the work of Albrecht Dürer (Figure 2(d)).

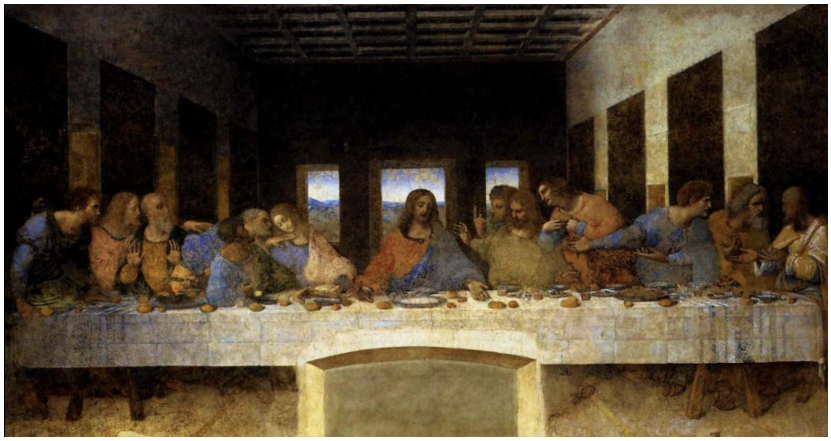
2.3 Early modern period

One-point perspective allows the artist to realistically depict lines which run parallel to the line of the viewer's gaze or which are at a right angle to it. That is, it serves to represent rectangular objects which the viewer faces straight-on. For objects defined by lines at an oblique angle to the gaze of the viewer,

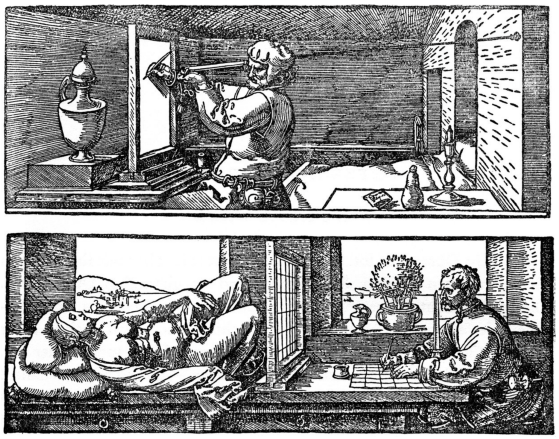
(a) *The School of Athens*, Rafael (1509–1511)



(b) *The Healing of the Cripple and Raising of Tabitha*, Masolino (1426–1427)

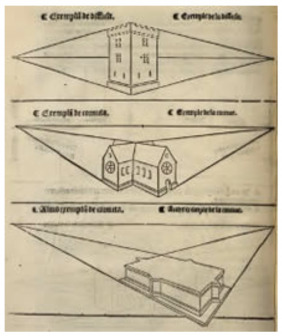


(c) *The Last Supper*, Leonardo da Vinci (1492-1498)

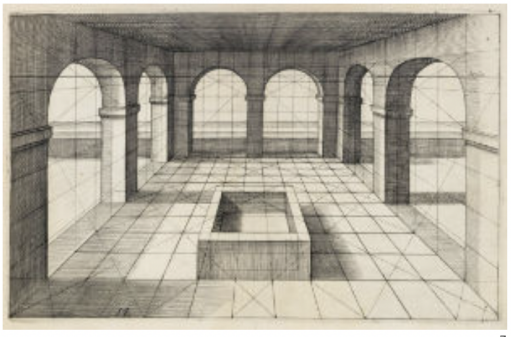


(d) Woodcuts by Albrecht Dürer (1525)

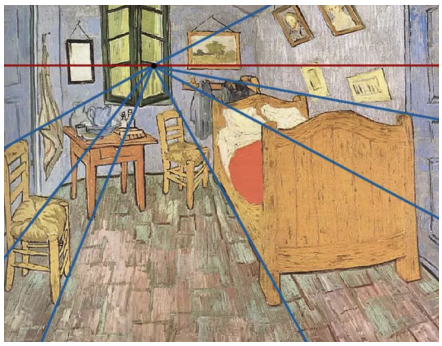
Figure 2: Examples of one-point perspective from the Rennaissance period



(a) *De Artificiali perspectiva*, Jean Pélérin (1505).



(b) *Perspective, c'est a dire, [...]*, Vredeman de Vries, The Hague (1604–1605).



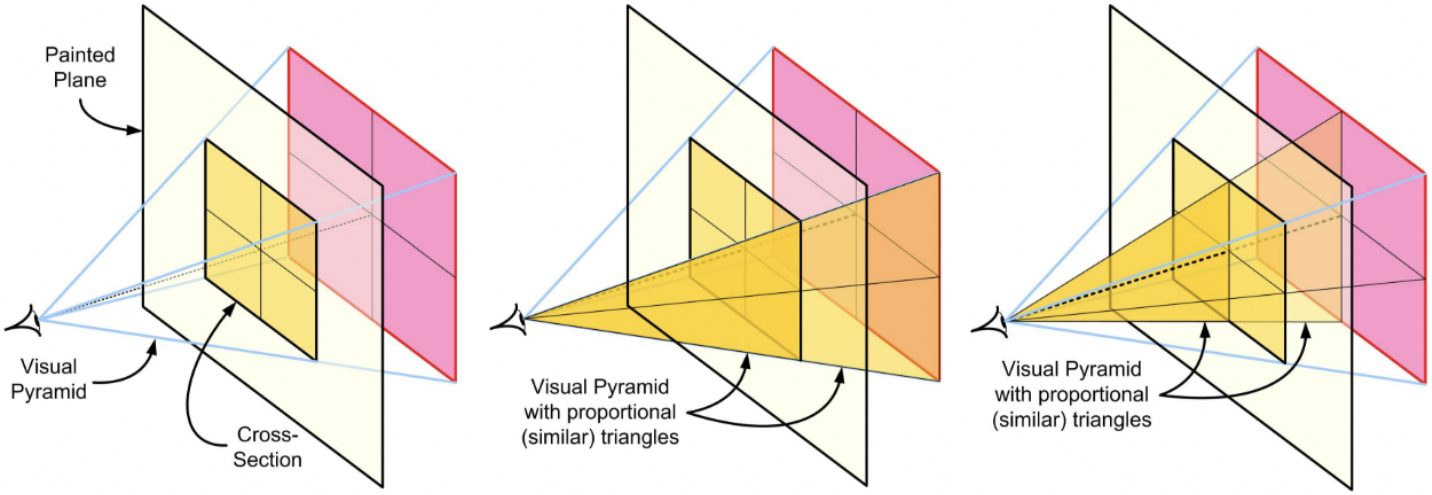
(c) *The Bedroom in Arles*, Vincent van Gogh (1888).

Figure 3: Examples of linear perspective from the early modern and modern periods

one requires *two-point perspective*. The method of two-point perspective first appears in the 1505 work *De Artificiali Perspectiva* by French diplomat and mathematician Jean Pélérin. A full treatment of the geometric methods of perspective which existed by the early 16th century was provided in 1560 by French artist Jean Cousin. Since that time artistic training in Europe, and eventually throughout the world, has assumed linear perspective as a fundamental skill for aspiring artists.

3 Alberti's Method

In his 1435 manuscript, Alberti introduces geometry, specifically the idea of **similar triangles**, to quantify the sizes of the figures and objects in a painting, as they are transformed by the perspective map (taking 3D objects into 2D paintings). He uses two important constructs for his perspective computations, namely the **visual pyramid** and **cross-section.**



The following quotes are from John Spencer's translation of Book 1 (originally in Latin) into English:

The [visual] pyramid is a figure of a body from whose base straight lines are drawn upward, terminating in a single point. The base of this pyramid is a plane which is seen. The sides of the pyramid are those rays which I have called extrinsic. The cuspid, that is the point of the pyramid, is located within the eye where the angle of the quantity is. [pg.47-48]

Now, since in a single glance not only one plane but several are seen, we will investigate in what way many conjoined [planes] are seen. [pg.51] [...] *Where this is a single plane, either a wall or a panel on which the painter attempts to depict several planes comprised in the visual pyramid, it would be useful to cut through this pyramid in some definite place, so the painter would be able to express in painting similar outlines and colours with his lines. He who looks at a picture, done as I have described [above], will see a certain cross-section of a visual pyramid, artificially represented with lines and colours on a certain plane according to a given distance, centre and lights. Now, since we have said that the picture is a cross-section of the pyramid we ought to investigate what importance this cross-section has for us.* [p.52]

Let us add the axiom of the mathematicians where it is proved that if a straight line cuts two sides of a triangle, and if this line which forms a triangle is parallel to a side of the first and greater triangle, certainly this lesser triangle will be proportional to the greater. [p.52]

Now let us translate our thinking to the pyramid. We should be persuaded that no quantities equidistant to the cross-section can make any alteration in the picture, because they are similar to their proportionates in every equidistant intercision. From this it follows that when the quantity with which the outline is constructed is not changed, there will be no alteration of the same outline in the picture. It is now manifest that every cross-section of the visual pyramid which is equidistant to the plane of the thing seen will be proportional to that observed plane. [pp.53-54]

4 Project

Put things in perspective! Practice drawing objects in perspective using the provided worksheets. Tell me what image (object, location, etc.) you'd like to draw for your class project, and I will bring a picture of it for you next time to draw in your sketch book!