What is an algorithm?

Art of Mathematics, Summer 2023

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1 Introduction

Algorithms are some of the most fundamental objects in mathematics, and under various different names, in our lives. An algorithm is simply a list of specific instructions to solve a problem or complete a task.

For example, a baking recipe could be considered an algorithm. At the risk of being a little ridiculous, if we wrote it mathematically, this is what an "algorithm" for chocolate chip cookies might look like:

Algorithm 1 Chocolate Chip Cookie Algorithm

Require: 1/2 cup butter, 1/2 cup granulated sugar, 1/2 cup brown sugar, 2 tsp vanilla extract, 1 egg, 1 3/4 cup flour, 1/2 tsp baking soda, 1/2 tsp baking powder, 1 cup chocolate chips
Ensure: 24 delicious chocolate chip cookies
Preheat oven to 350 degrees Fahrenheit.
In a large bowl, mix butter and sugars well until smooth.
Stir in vanilla extract and egg until just incorporated.
In a separate bowl, whisk together dry ingredients: flour, salt, baking powder, and baking soda.
while dry ingredients remain **do**
    Mix 1/4 cup of dry ingredients into butter mixture until smooth.
end while
Fold chocolate chips into dough
for cookie = 1, 2, ..., 24 **do**
    Scoop 1.5 tbsp of dough for cookie and place 2" from other cookies on baking sheet
end for
Bake for 10 minutes.

An algorithm consists of three main parts: inputs (the ingredients), outputs (the finished product), and steps (the instructions). Everyone who implements an algorithm using the same inputs should get the same output every time. In other words, if you and I have the exact same ingredients and follow the exact same recipe, we should get identical chocolate chip cookies.

2 Algorithms and Art

We'll look at several different art forms with algorithmic influences, including tilings, weavings, and paintings.

2.1 Mosaic Tiles

Algorithms have been used in art throughout history, but one of the earliest and most beautiful examples is *girih* tiling. Girih tiles, Persian for "knot", form a collection of five patterns that have been used in Islamic architecture since the 11th century, fit together according to certain specifications. Templates for the tiles were found on the Topkapi Scroll in the Topkapi Palace in Istanbul, originally consulted by artisans to fabricate the tiles and architects to fit them together during the time of the Ottoman Empire.

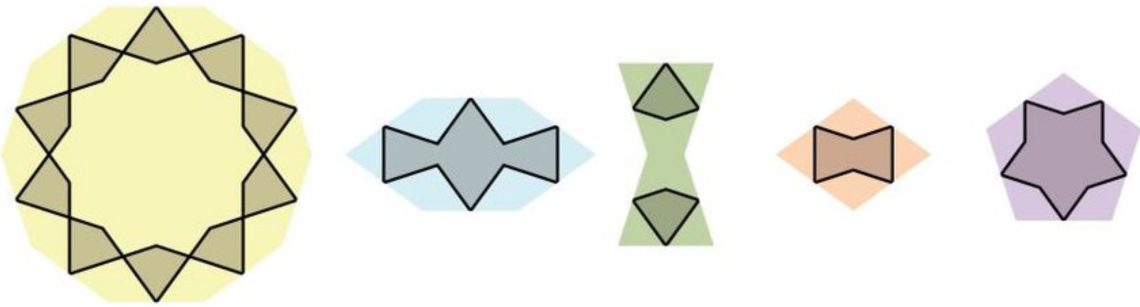


Figure 1: The 5 classes of girih tiles: from left to right (1) a regular dodecahedron, (2) a convex hexagon, (3) a non-convex hexagon, or "bow-tie", (4) a rhombus, and (5) a regular pentagon. Image courtesy of Lars S. Eriksson.

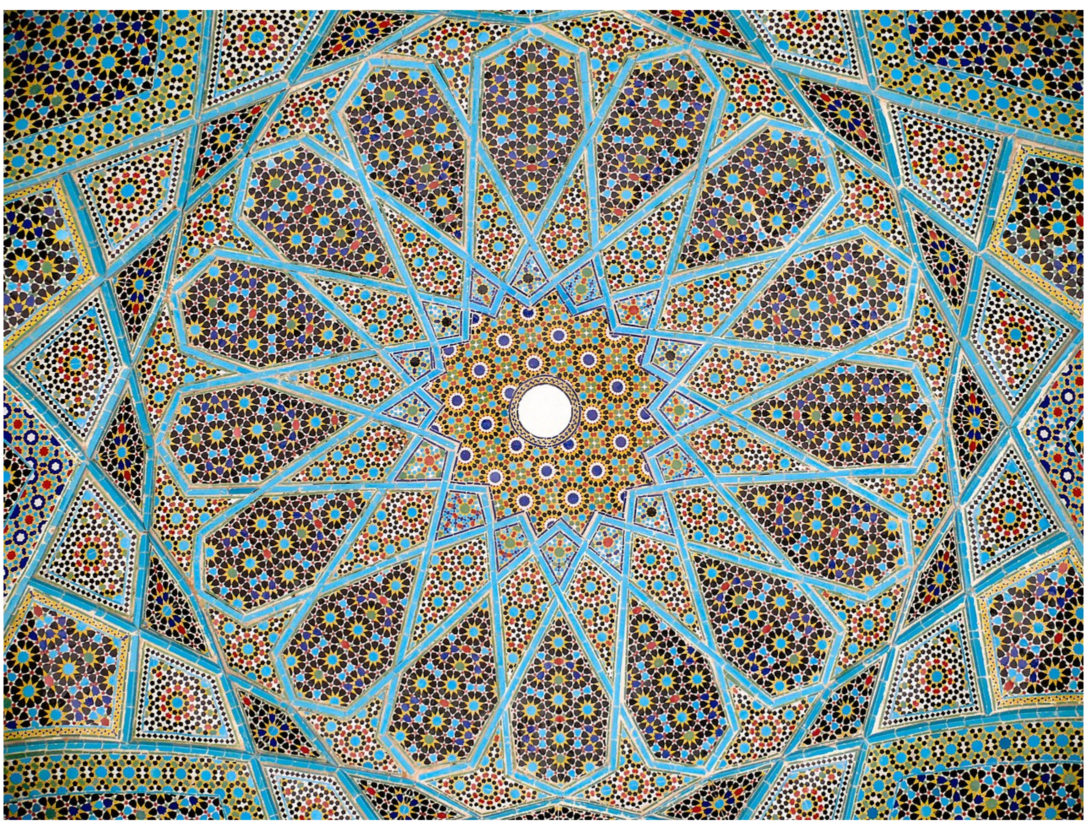


Figure 2: Girih tile patterns, from the ceiling of the Tomb of Hafez in Shiraz, Iran. Wikipedia.

2.2 Woven Arts

There is a long historical connection between textiles and mathematics. Algorithms for weaving are complex and often passed down through generations, such as in the Andean weaving tradition dating back centuries.

In the Pacchanta village of Peru, the designs, or *pallay* (Quechua for "to pick") are constructed one thread at a time. Young girls are taught the thread pick-up patterns in numbers, *hoq* (one), *iskay* (two), *kinsa* (three), *tawa* (four), etc., as they memorize the mathematical relationships in the patterns.

Figure 3: A women's shoulder cloth, or *lliclla*, featuring white beads (*pini*), zig-zag trim (*qenqo*), and sequins, to mimic sunlight shimmering off a lake. Andrea M. Heckman, Smithsonianmag.org.

2.3 Renaissance Art

We will talk about these techniques in more detail when we talk about proportion and perspective, but Renaissance artists utilized algorithms based on the so-called vanishing point of an image to make their paintings geometrically and visually "correct" from the perspective of the viewer.

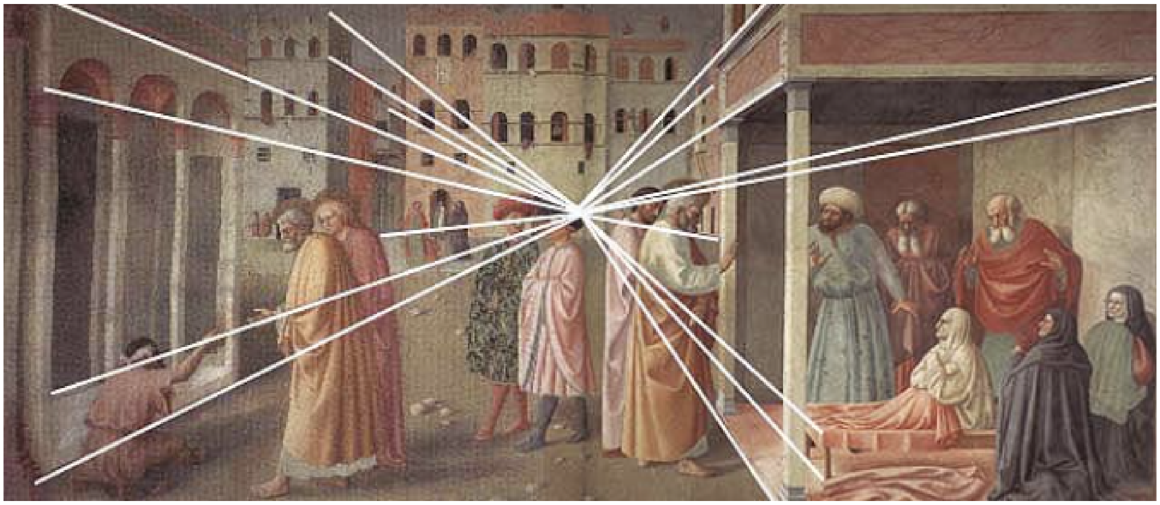


Figure 4: 'The Healing of the Cripple and the Raising of Tabitha', by Masolino (1425). Observe how all of the painting's structures, from the very front of the canvas to the very distant background, all converge to the intersection point at the center of the painting. Image courtesy of Christopher W. Tyler, "Perspective as a Geometric Tool that Launched the Renaissance."

2.4 Modern Art and Technology

"Many of [M.C. Escher's] artworks are structured in such a way that it is possible to write algorithms (computer programs) that generate them. This was indeed done by several authors from the computer science field. One example is Michael Trott who in his book 'Mathematica Guidebook for Graphics' constructed a computer program that generates image of the Escher's well-known lithograph *Reptiles*." - Vlatko Ceric, *Algorithms, Mathematics, and Art*, SNE Simulation Notes Europe, 2011.

Figure 5: From Wikipedia: "*Reptiles* depicts a desk upon which is a two dimensional drawing of a tesselated pattern of reptiles and hexagons, Escher's 1939 *Regular Division of the Plane*. The reptiles at one edge of the drawing emerge into three dimensional reality, come to life and appear to crawl over a series of symbolic objects (a book on nature, a geometer's triangle, a three dimensional dodecahedron, a pewter bowl containing a box of matches and a box of cigarettes) to eventually re-enter the drawing at its opposite edge. Other objects on the desk are a potted cactus and yucca, a ceramic flask with a cork stopper next to a small glass of liquid, a book of JOB cigarette rolling papers, and an open handwritten note book of many pages. Although only the size of small lizards, the reptiles have protruding crocodile-like fangs, and the one atop the dodecahedron has a dragon-like puff of smoke billowing from its nostrils."

Technology today has also made it possible to generate art from mathematical equations using computers. CAD (computer-aided design) is used in almost every industrial production facility, relying on equations to precisely measure, cut, and shape materials.

Computers can also generate art directly from equations. For instance, fractals are a popular subject of mathematics and paintings–geometric shapes that appear similar at arbitrarily small scales, as illustrated in the figure showing successive magnifications of the Mandelbrot set. This feature of fractals is called self-similarity, also called expanding symmetry or unfolding symmetry.

The Mandelbrot set is generated by just two innocent-looking equations:

$$z_0 = c \tag{1}$$
$$z_n = z_{n-1}^2 + c, \tag{2}$$

where c is any complex number, and $n = 1, 2, 3, ...$ is any positive integer. If $|z_n|$ does not shoot off to infinity as n goes to infinity, then c is in the Mandelbrot set.

The points that belong to the Mandelbrot set are usually colored black. The points that don't belong to the Mandelbrot set will leave the circle with radius 2 for some n; this is called the escape-time. Letting each point get a color that depends on its escape-time, you can make patterns that infinitely repeat themselves when zooming in. (Not exactly identical but almost). The black blobs are all similar to each other but are actually different.

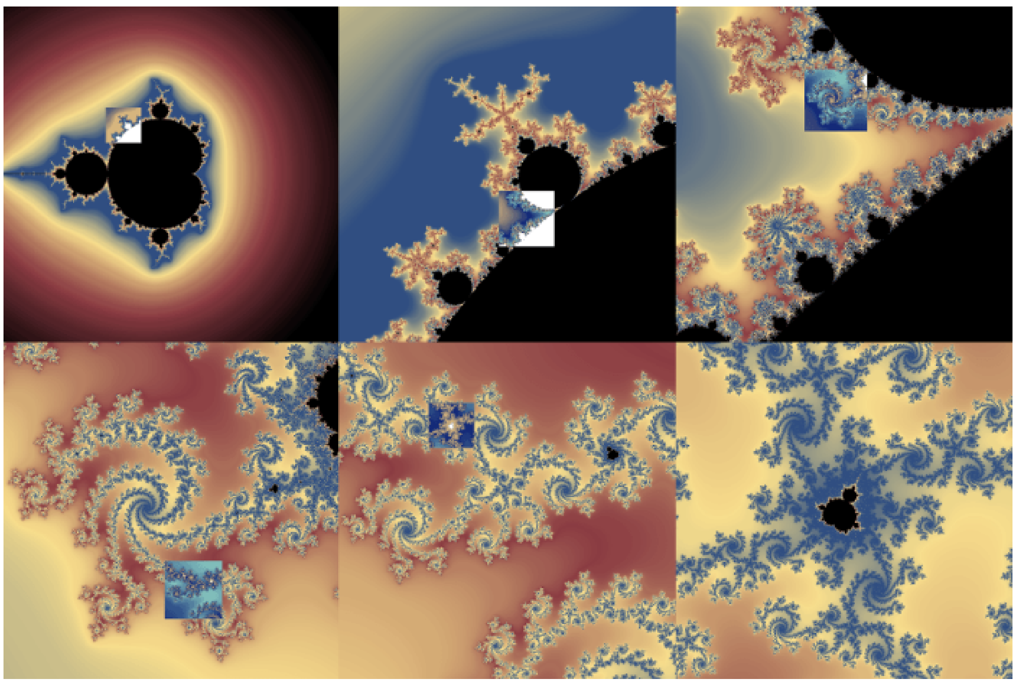


Figure 6: Successive zooming-in on the Mandelbrot set; next zoomed-in image appears from left-to-right in the smaller rectangle. Image courtesy of "The Mandelbrot Set" by Malin Christersson.

3 Projects: Paper Arts

Create your own origami and write the algorithms for someone else to create them.

Bonus if you ask someone else to try out your algorithm and see what design they get. How could you improve your algorithm's clarity for easier interpretation?

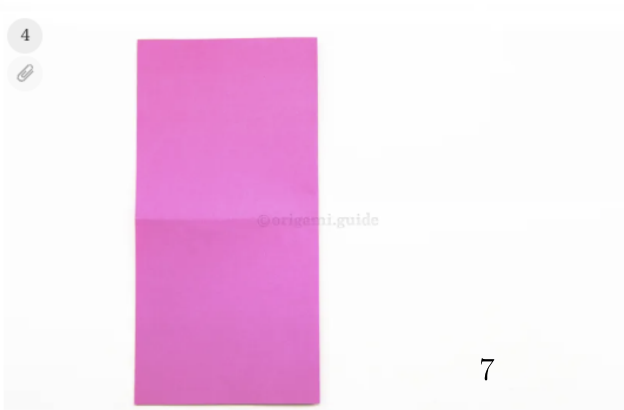
Here is an example of origami with an accompanying "algorithm" for a lotus flower for inspiration.

Suggestion: use a lot of descriptive words like "fold the *top right* corner to the *midpoint* of the *left edge*," etc. Imagine you are describing how to do it to someone over the phone who will take you very literally.

1. This is the back of the origami paper, usually white. This colour will be visible on the bottom of the lotus if you fold the last set of petals.

2. Fold the bottom edge of the paper up to the top edge.

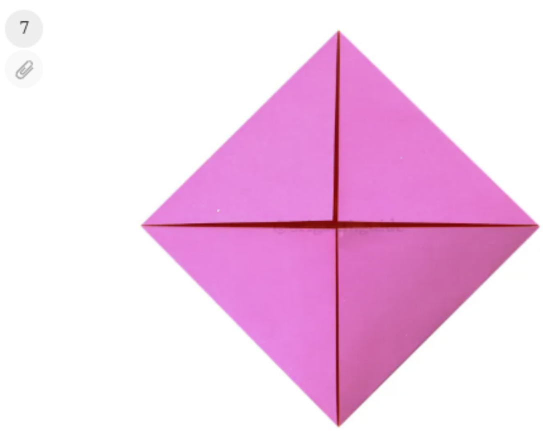
3. Unfold the paper.



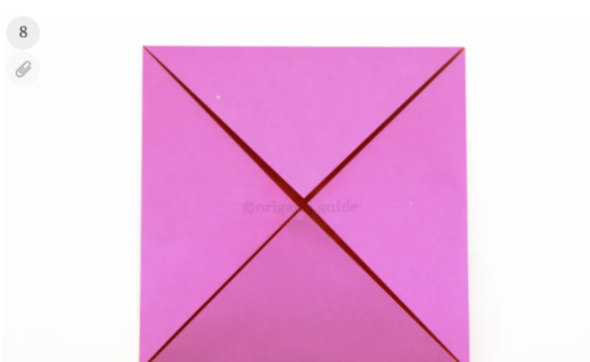
4. Fold the right edge over to the left edge.

5. Unfold the paper.

6. Fold the bottom right corner to the middle of the paper.

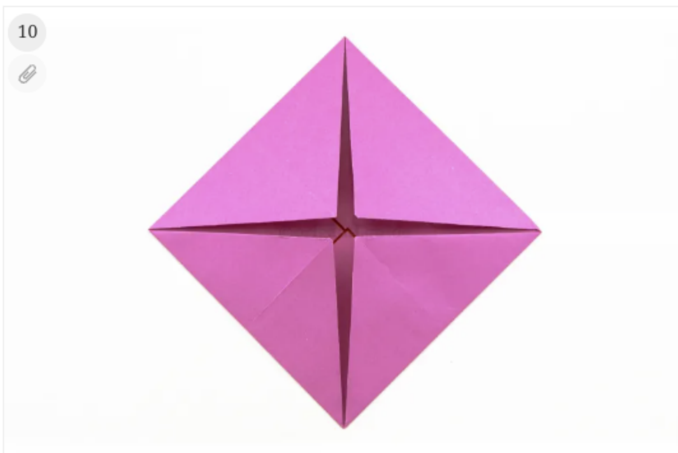


7. Fold the other three corners of the paper to the middle of the paper.



8. Rotate the paper.

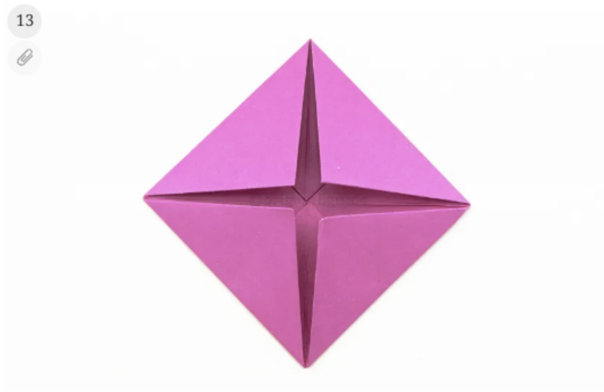
9. For a second time, fold one corner to the middle of the square.



10. Fold the other three corners to the middle of the square.

11. Rotate the paper.

12. For the third time, fold a corner into the middle of the square.



13. Fold the other three flaps to the middle.

14. Flip the model over to the other side.

15. Fold the bottom point to the center.

16. Fold the other three points to the center.

17. Fold one of the corners in a little.

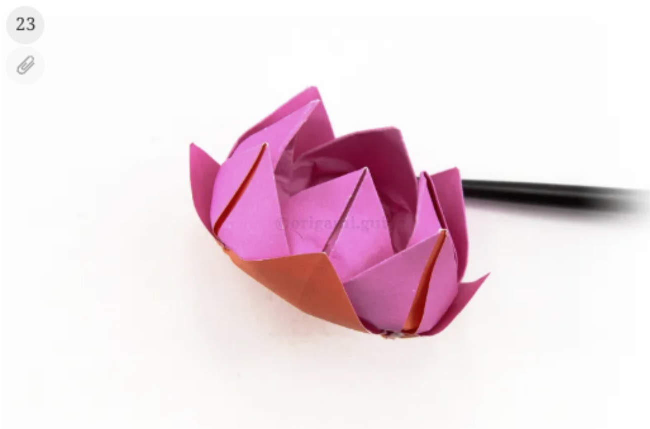
18. Whilst holding the folded corner with one hand, bring the flap behind it forwards, reversing it over to this side.

19. Fold the other three corners in the same way.

20. Fold the little inner flaps up inside the flower.

21. Flip the flower over to the other side. Peel the flaps carefully over to the other side.

22. The flaps can be fully reversed over the top of the first set of petals.



23. Optionally fold the last set of flaps from the back of the lotus.

24. The finished origami lotus flower.